

Modeling the Response of Dielectric Slabs on Ground Planes Using CW Focused Millimeter Waves

Mahdiar Sadeghi^{1*}, Elizabeth Wig¹, Ann Morgenthaler¹, Carey Rappaport¹

¹ (Affiliation): Electrical and Computer Engineering Department, Northeastern University, Boston, MA 02115

*sadeghi.ma@husky.neu.edu

Abstract—We present a novel non-iterative model based on ray analysis to characterize non-metallic, weak dielectric objects (like threat objects) on the surface of a highly conducting background (like the human body) using a focused continuous millimeter-wave sensor. For a simple constant thickness dielectric slab on a ground plane, there are five primary scattering phenomena that must be considered.

Index Terms—dielectric scattering, focused sensing, inverse scattering.

I. INTRODUCTION

Detection of weak dielectric threat objects is a challenge for millimeter-wave scanning systems. This paper describes a novel, non-iterative, analytic method to characterize the thickness and refractive index of a dielectric slab using a continuous-wave (CW) radar system with a Fresnel reflector aperture [1]. The system focuses the signal on a set of focal points in 3D space and measures the return signal.

The radar system can be modeled in two dimensions by an array of point sources with adjusted phase along a horizontal line above the ground plane on which the dielectric sits. It is assumed that the array has the same phase adjustment on transmission and reception, and that the dielectric is ideal and has unknown thickness and permittivity.

When a focused wave is sent from the transmitter array toward the dielectric slab, it will scatter from two interfaces: the ground plane interface at the dielectric bottom surface and the dielectric/air interface at the dielectric top surface. Neglecting additional scattered fields inside the dielectric, five dominant scattering phenomena emerge from this model with three material constant interfaces: at the dielectric top surface, at the ground plane, and at the dielectric image surface; and their associated reflections.

This paper aims to describe the signal resulting from the addition of these phenomena over a one-dimensional distribution of focal points, and to use it to characterize the dielectric slab.

II. METHODOLOGY

Fig. 1 shows a slab of dielectric of thickness d on top of a metal ground plane and a horizontal transmitter/receiver array located at a distance H above the ground; the transmitter array focuses to different depths into the air and the dielectric. Considering the image of the dielectric and antenna array relative to the metal ground plane, there will be three scattering interfaces.

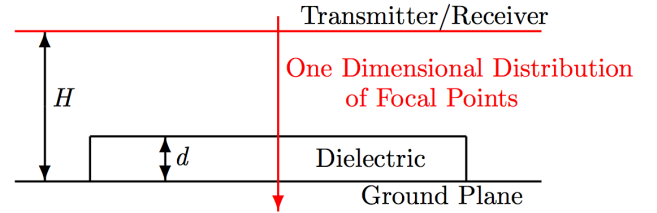


Figure 1. Schematic of the Problem

The novel aspect of this work is that by limiting the scattering at interfaces to single points along the range axis generated using a focused one-dimensional array, we can straightforwardly predict what the received signal should look like from a given dielectric thickness and refractive index. Considering only the first reflection from the dielectric top surface or its image, Fig. 2 shows the five scattering phenomena to be considered.

In the first case, we assume that scattering occurs at the top surface of the dielectric. The rays scatter back to the receiving array (Case 1A) and forward into the dielectric (Case 1B). In Case 1A, the path travels only through the air, with the same path length on transmission and reception. This received signal is scaled by the scattering cross-section, approximated by the reflection coefficient of the dielectric, about -0.25.

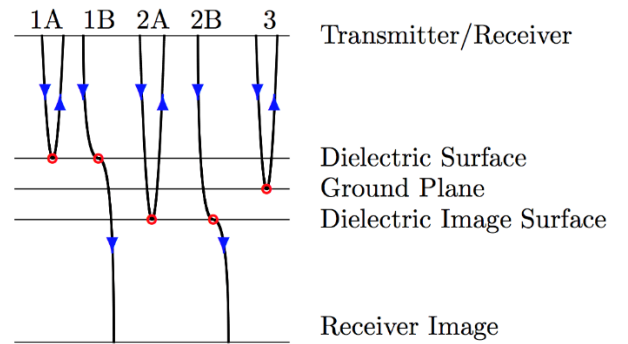


Figure 2. Scattering Phenomena Mechanisms

In Case 1B, using image theory to analyze the ray paths, the images of the dielectric slab and receiving array replace the ground plane. The transmitted rays now begin at the source, scatter at the dielectric/air interface, pass through the double wide dielectric slab, refract from inside at the image interface (with intensity multiplied by the transmission coefficient), and finally point to the receiver array image.

In the second case, the scatterer is located at the image surface of the dielectric; rays refract through the true dielectric surface to scatter at this point. Again, contributions to the signal come from both backscatter (Case 2A) and forward scattering (Case 2B) at the image interface. In Case 2A, the backscattered rays refract back through the surface and return to the true receiver array; in Case 2B the forward scattered rays travel through air to the image receiver array. Note that the scattering at the image of the dielectric/air interface has a coefficient based on the material contrast that is equal and opposite to the scattering coefficient of Case 1A above.

With a scatterer at the ground plane (Case 3), the signal reflects from the ground plane back to the receiver array with no image involved. The ray paths into and out of the dielectric are refracted, and the transmission coefficient must be applied for each refraction.

The coherent addition of all these signals (with appropriate choice of wavenumber k), for Cases 1A, 1B, 2A, 2B, and 3 – gives the final signal received by the array.

For Cases 1A, 2A, and 3, the backscattered paths are the same as the incident paths, so one would expect strong responses for these scattering phenomena. For Cases 1B and 2B, the forward scattered paths are different from the incident paths, so these cases will be relatively incoherent and unfocused for the sensing system.

Mathematically, there are several path lengths to compute for each contribution to the signal. The first (r_1 in Fig. 3) is the distance from a particular transmitter through the air to the surface of the dielectric at $(0, d)$.

$$r_1 = \sqrt{x_i^2 + h^2} \quad (1)$$

where x_i is the horizontal distance from a particular transmitter to the central axis, $h = H.d$ is the distance from the transmitter array to the dielectric slab, and d is the thickness of the slab.

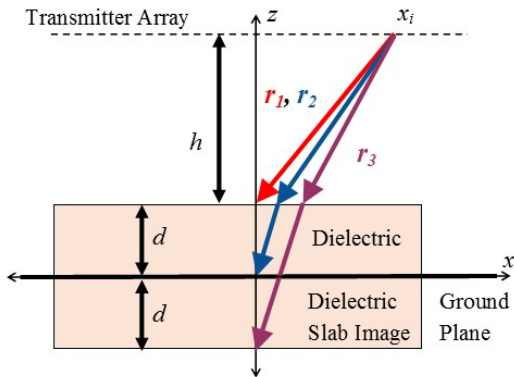


Figure 3. Ray Path Calculations

In Fig. 3, r_2 and r_3 are the reflected paths of rays when the array is configured as if to focus at points inside of the dielectric. The rays refract to hit the ground plane and image surface at distances d and $2d$ below the top dielectric surface.

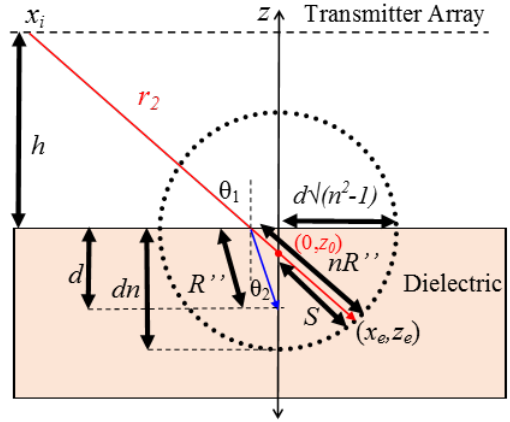


Figure 4. Wave-front Calculations

The electric path length traveled inside the dielectric (the blue path in Fig. 4) is given by nR'' , where R'' is the physical distance traveled inside the dielectric and n is the refractive index of the dielectric. This electrical distance is equivalent to the physical distance nR'' for a ray continuing through air instead, shown by the red arrow extending below $z=d$.

The extended ray ends at a point (x_e, z_e) . For rays starting from different points x_i , there are different (x_e, z_e) points, but all of these points lie on an ellipse given by [2]:

$$\left(\frac{x_e}{d\sqrt{n^2-1}}\right)^2 + \left(\frac{z_e}{dn}\right)^2 = 1 \quad (2)$$

The path length can be found by taking the distance from (x_i, h) to (x_e, z_e) :

$$r_2 = \sqrt{(x_i - x_e)^2 + (h - z_e)^2} \quad (3)$$

Moreover, the line between the two points (x_i, h) and (x_e, z_e) is normal to the ellipse, implying:

$$x_i = x_e + (h - z_e) \frac{n^2 x_e}{(n^2 - 1) z_e} \quad (4)$$

Substituting x_i from equation (4) and x_e from equation (2) into the path equation and simplifying, gives:

$$r_2 = \frac{(h - z_e)}{z_e} \sqrt{\frac{d^2 n^4 - z_e^2}{n^2 - 1}} \quad (5)$$

The values of r_2 and x_i in (4) and (5) can be plotted parametrically against each other in terms of z_e . This complicated relationship between r and x_i can be approximated very closely by a simpler equation for path in terms of x :

$$r_2 \approx \sqrt{x_i^2 + (h - z_0)^2} + S \quad (6)$$

where S and z_0 are constants: $-z_0$ is the depth below the air/dielectric interface corresponding to the array focal point in air which produces the true focus in dielectric at the depth d , and S is the phase offset that compensates for the extra path length to the more distant focal point. This approximation can

be made by choosing two points along the parametric curve of x_i and r_2 (for example, $z_e = -dn$ and $z_e = -pdn$, where p is the fitting parameter between 0 and 1), inserting them into (6), and solving for z_0 and S . These values for S and z_0 can be substituted into equation (6) to find the approximate path length formula as a function only of antenna position x_i :

$$S = -1/2 \frac{d(-1+p)(2hn+dp(1+p))}{-p(h+dn)+(h+pdn)\sqrt{\frac{(n^2-p^2)}{(-1+n^2)}}} \quad (7)$$

and

$$z_0 = -dn + S \quad (8)$$

To find r_3 , the path length to the image dielectric surface, the ellipse method can again be used, substituting $2d$ for d , as the dielectric and its image form a slab with twice the thickness.

For example, for array distance $h = 80$ cm, array width $2x_i = 100$ cm, dielectric slab thickness $d = 3.7$ cm, and index of refraction $n = 1.73$, the fitting parameter can be chosen to be $p = 0.92$. The parametric equation in terms of z_e and the approximation in terms of $S = 4.34$ and $z_0 = -2.05$ match almost exactly in the desired range of x_i . The error of the approximation (Fig. 5) in this range is less than 0.003 cm.

Therefore, using (6), the path lengths r_2 and r_3 can be found simply and accurately in terms of the antenna array position. It is fortuitous that the form of (6) is similar to the form for rays focused in air from a linear array. The signal from each component can be calculated given these path lengths. For the various values of r (r_1 , r_2 , or r_3) the corresponding focused sum S_1 , S_2 , S_3 is given by:

$$S_n = \sum_i^{\text{#antenna elements}} e^{-jkr_{ni}} * e^{j\Phi_i} \quad (9)$$

where k represents the wavenumber and Φ represents the phase for the i^{th} antenna element.

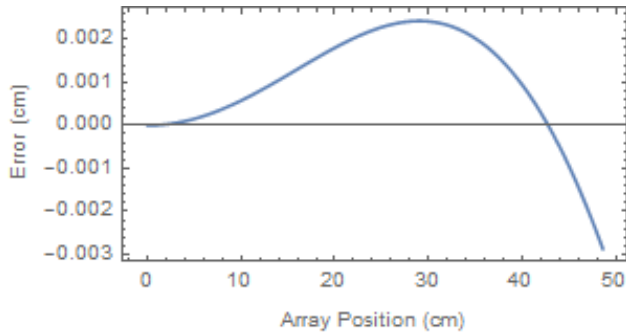


Figure 5. Error of path length approximation, for $h = 80$ cm, maximum $x_i = 50$ cm, $d = 3.7$ cm, and $n = 1.73$

The total signal from each scattering component can be found by multiplying the values of S_n on transmission and reception for each scattering case with the appropriate transmission and scattering coefficients:

$$\begin{bmatrix} E_{1a} \\ E_{1b} \\ E_{2a} \\ E_{2a} \\ E_3 \end{bmatrix} = \begin{bmatrix} \Gamma * S_1 * S_1 \\ \Gamma * T_2 * S_1 * S_3 \\ T_1 * (-\Gamma) * T_2 * S_3 * S_3 \\ T_1 * (-\Gamma) * S_3 * S_1 \\ T_1 * T_2 * S_2 * S_2 \end{bmatrix} \quad (10)$$

where Γ is the scattering coefficient of the dielectric, and $T_1 = 1 + \Gamma_1$ and $T_2 = 1 + (-\Gamma)$ are the transmission coefficients. Scattering from air into dielectric multiplies by a coefficient of Γ ; scattering from inside the dielectric to air multiplies by $-\Gamma$; refraction into the dielectric multiplies by a coefficient of T_1 , while refraction out of the dielectric multiplies by T_2 .

The most speculative scattering assumption being made with this ray analysis is that the scattering from an interface occurs as if all incident rays were focused at a point on the interface, even when the focus point might be nearby but not exactly on the interface. The scattering from a point scatterer is straightforward to model with rays, and with a focused illumination with all rays converging on a single point on an interface, this is an effective approximation. Once the focal point leaves the surface, though, the illuminated region of the surface is no longer a small point, and the scattering from the surface is no longer that of a point scatterer. However, if the focal point is not too far from the surface, the rays will still converge, approaching each other with small separation, and their respective phases will be almost the same. The scattering from a single point on the interface will still reasonably represent the scattering from the partially focused spot. The intensity variation of the scattering with distance from the interface will be close to true values past the half-power point, although the side-lobe locations and levels will be inaccurately predicted. For the purposes of identifying locations and pulse widths of the reconstructed signals, this approximation is acceptable.

III. COMPUTATIONAL SIMULATION

In order to solve the problem computationally, a two-dimensional finite difference frequency domain (FDFD) method is used to generate field solutions in free space, dielectric half space, and dielectric layer on metal background. Computed fields are accurate to the second order in the spatial sampling interval [3]. Assuming an array of 153 point sources with appropriate phase and horizontal polarization as the excitation source, we simulate an aperture with 100 cm width located at 80 cm above the ground plane. The focused response is considered as a summation of the calculated scattered field at point sources multiplied by the transmit phase for each array element.

For a laptop computer with 16 GB memory, this simulation takes 12 hours for 25 focal points above and below the ground on the z -axis. Fig. 6 shows the total wave magnitude distributions near the focal region for Case 3, with array phase specified to focus at the bottom of the dielectric slab with relative permittivity of 3.0. Clearly, the maximum response in air occurs when the waves are focused at 1.6 cm ($z_0 = 21$ cm) above the nominal position of the ground plane

(Fig. 6(a)), while the real focusing through the dielectric at a depth of $-d$ is on the nominal ground plane (shown in Fig. 6(b) for a dielectric half space to remove scattering from the ground plane). Note the increased backscatter relative to the uniform air case shown in Fig. 6(a) above the dielectric interface at $z = 0.037$. Fig. 6(c) shows the total field intensity for the full geometry with 3.7 cm thick slab and ground plane. The backscatter in this case is significantly greater than for the dielectric alone. This is due to the strong reflection from the metal ground plane.

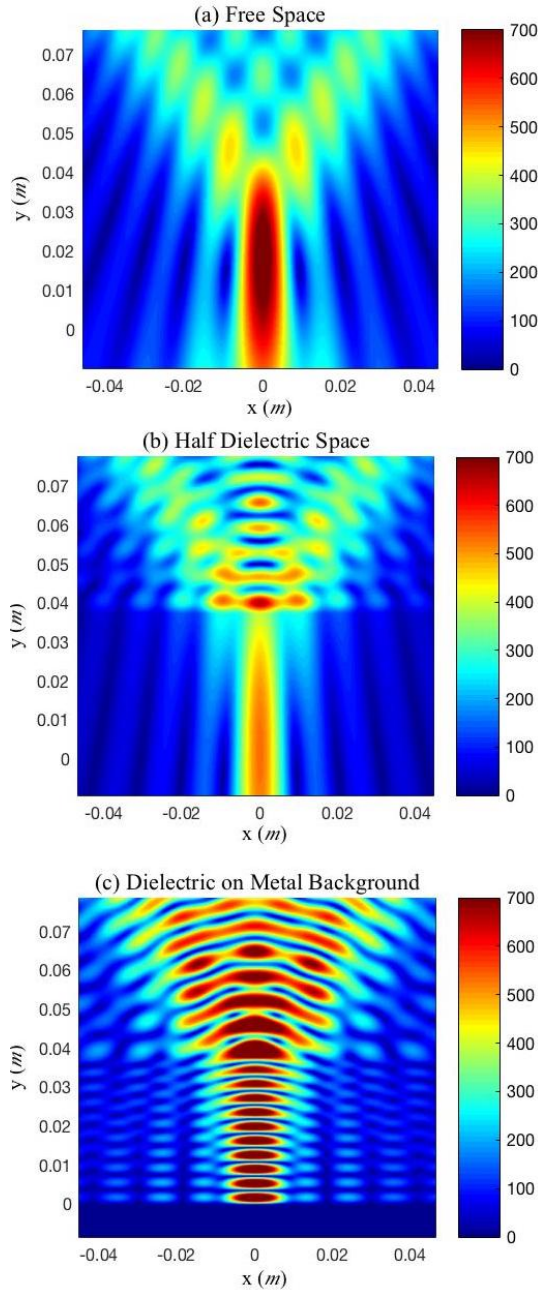


Figure 6. Total field magnitude simulations at 24.16 GHz for focusing in air at $z = 0.021$ m when the geometry is: (a) uniform air, (b) air and dielectric ($n = 1.73$) half space $z < 0.037$ m, and (c) air, dielectric 0.037 m thick slab over metal ground plane.

IV. RESULTS

The graphs plotted in Fig. 7 resulting from these mathematical models were compared to 2D FDFD simulation results and to data from a real eqo scanner, manufactured by Smiths Detection. The eqo system employs a large two-dimensional Fresnel reflector array of electronically activated scattering panels, which produces a tightly focused rapidly raster scanned spot [1].

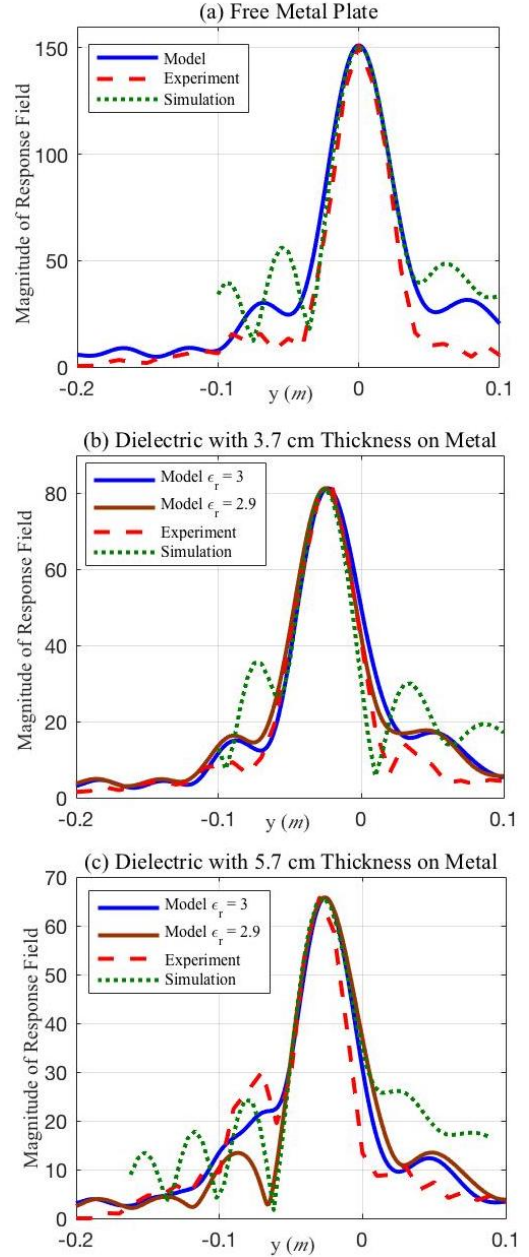


Figure 7. Comparison of the response versus range along the axis of symmetry of the ray model with $n = 1.73$ and 1.64 , the 2D FDFD simulation with $n = 1.73$, and the experimental measurements of Smith's eqo system at 24.16 GHz for (a) bare metal plate, (b) metal plate with affixed 3.7 cm thick dielectric slab, and (c) affixed 5.7 cm thick dielectric slab.

The 24.16 GHz CW focused spot scatters on or near the target surface, and then returns to the same array and the receiver co-located with the transmitter. The experimental system measured three cases: bare metal ground plane, and the ground plane with two affixed dielectric slabs (3.7 and 5.7 cm thick) with a relative permittivity of approximately 3, at a distance of approximately 80 cm from the array. Fig. 7(a) compares the modeled and simulated (blue and green) and measured (red) responses with the maximum magnitude of the model normalized to fit the measured value. Figs 7(b) and (c) plot the measured, FDFD simulation for $\epsilon' = 3.0$, and ray model responses for slabs with dielectric constants of both 3.0 (dark blue) and 2.9 (brown), showing the sensitivity of the model to slight changes in dielectric constant.

As can be seen, the pulse shapes and positions of each graph are very similar, with practically identical widths among the models of 4.7, 5, and 4 cm for the three different cases, showing the dependence of dielectric slab thickness. The sidelobe levels have the worst agreement, probably due to the simplified two-dimensional nature of the models. However, the main features of the responses – pulse position and width – are accurately predicted by both the computationally intense FDFD model and the extremely fast ray model.

V. CONCLUSION

Based on ray tracing analysis using the five novel scattering phenomena identified in this analysis, a fast, non-iterative model for determining the response of focused sensing of a dielectric slab on a metal ground plane is presented and validated with both computational simulation and real experiments. This analysis is useful for characterizing lossless dielectric objects on a conductive background for concealed object detection applications using continuous millimeter-wave focusing radar. Characterization of tilted or arbitrary-shaped dielectric objects can be included in future work.

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